

Problem Set 1

1. Griffiths 9.11.

2. Griffiths 9.18.

3. Griffiths 9.19.

4. Griffiths 9.20.

5. At the rate of 1 card/sec, psychic Uri Geller (<http://skepdic.com/geller.html>) turns over each card in a deck. He communicates by “paranormal” means the identity of each card to his assistant, from whom he is shielded with respect to sound and visible light.

As a physicist, you consider all EM waves to be normal. To test the notion that Uri’s talents defy the laws of physics, you resolve to design a shield that will prevent Uri from using any relevant EM frequency to communicate with his assistant.

(a) Roughly what minimum EM frequency must Uri use? (*Hint:* Consider that a 56 kbps modem operates over audio telephone frequencies.)

(b) Design a spherical shell, enclosing a volume of 1 m³ for Uri’s comfort, that will attenuate the EM waves generated by Uri’s brain to $\approx 1/400 \approx e^{-6}$ of their original amplitude. Use the minimum EM frequency that you calculated in (a).

(c) How much does your shield weigh? (Try to design the lightest shield that will do the job. Does it help to use a ferromagnetic material?)

6. An electromagnetic cavity can be considered to be just another resonant oscillator, with a quality factor Q equal to the ratio of the energy stored to the energy dissipated during the time interval $\Delta t = 1/\omega_0$. Consider a cubical box of side d whose inner surfaces are plated with an adequate thickness of silver, which is an excellent conductor. This cavity has a fundamental resonant angular frequency equal to

$$\omega_0 = \frac{c}{d} \times \pi\sqrt{2} ,$$

where the first factor can be identified from purely dimensional arguments, and the second factor, a function of the cavity’s geometry, is of order unity. Apart from a different geometrical factor of order unity, the Q of this cavity turns out to be of order

$$Q \approx \frac{V}{A\kappa^{-1}} ,$$

where V is the cavity’s volume, A is its inside surface area, and κ^{-1} is the skin depth. Thus, Q is of the same order as the ratio of the cavity’s volume to its “skin depth volume”.

(a) Taking $d = 10$ cm, what Q can be achieved?

(b) If the cavity is kept at the same size, would it help to operate it at one of its higher frequency modes?

(c) If the cavity is always operated at its fundamental frequency, would it help to build it bigger?

7. Show that the results in Griffiths Eq. (9.147) are equivalent to the familiar formulæ

$$\begin{aligned} R &= \frac{Z_2 - Z_1}{Z_2 + Z_1} \\ T &= \frac{2Z_2}{Z_2 + Z_1} , \text{ where} \\ Z &\equiv \frac{\tilde{E}_0}{\tilde{H}_0} , \\ R &\equiv \frac{\tilde{E}_{0R}}{\tilde{E}_{0I}} , \text{ and} \\ T &\equiv \frac{\tilde{E}_{0T}}{\tilde{E}_{0I}} , \end{aligned}$$

and where Z is the characteristic impedance of the medium, R is the amplitude reflection coefficient, and T is the amplitude transmission coefficient.

8. Consider a dilute material with $\epsilon = \epsilon_0$ and $\mu = \mu_0$, but with slight conductivity $\sigma = \beta\epsilon_0\omega$, where $\beta \ll 1$ is a constant. EM radiation of angular frequency ω is normally incident from vacuum upon this material.

(a) Relative to the incident field, show that the reflected electric field has a magnitude of $\beta/4$ and a phase shift of 90° .

(b) Show that the transmitted wave is attenuated with a skin depth equal to $\lambda_0/2\pi$ divided by β , where λ_0 is the vacuum wavelength, and that its \mathbf{H} lags \mathbf{E} by a phase shift equal to $\beta/2$.